criticism of the explanation given by Bolotin for the instability of columns having a tip mass under a tangential force and his own contribution to this problem.

Flax's analysis has removed many ambiguous points, for which I would like to thank him. I also agree with his views in general, but I differ in some points, which I would like to point out.

1) Although the closeness between the two values 20.05 and 20.19 seems to be a coincidence, the agreement between 20.19 and the buckling load of a fixed-pinned Euler column under conservation dead load is not insignificant.[1] In fact, it can be shown that this will hold true for a wide class of massless structures with a tip concentrated mass under follower forces. Flax' own analysis is a step in this direction. A good example for this is the well-known Leipholz problem. There are, of course, two Leipholz problems.[4] The first is the original problem[2] with distributed mass, where he found that the critical value is $P^c = 40.7\ EI/\ell^3$. The second is the corresponding massless problem with a tip concentrated mass. The critical load for this structure $P^c \cong 53.75\ EI/\ell^3$ was probably quoted for the first time by the author in Ref. 1. This value is also the critical value of a fixed-pinned Euler column. For this reason it seems, at least in some sense, to be a tautology to say that "in this special case the dynamical buckling is predictable from the static criteria," as stated by Flax, or "that a statical method could be used to find the buckling load of a nonconservative system," as stated by the author.

The author feels that it is misleading to speak of realistic models in the field of nonconservative follower forces. If we are ignoring effects like damping and nonlinearity, then we must regard the work done in this field as being mainly of theoretical interest. The "method" of transformed conservative statical system (T.C.S.S.) may be of some use in this sense.

3) It is interesting to see how a result thought to be well known and understood is still open to controversy, as in the present case. I think that many questions are still open in this respect, especially those concerning the postcritical behavior of nonconservative systems, and why, in the particular problem considered by the author, the conclusion of imperfection insensitivity agrees with the nonlinear dynamical analysis of Burgess,[3] which indicates soft flutter.

4) In conclusion, the author would like to make a few comments to the references cited by Flax. First, with regard to Ref. 3, it merely quotes the results of Pflüger without any analysis. The work of Pflüger does not deal explicitly with massless structures and is misleading in the interpretation that (m) must become infinity in order to represent a constraint leading to static buckling. In fact, any small mass different from zero would lead to the same result. However, having said that, the author painfully admits that he overlooked the undoubted fact that Bolotin obtained the critical value $P^c = 20.19\ EI/\ell^3$ long ago in his classic book. Nevertheless, in view of the deeper insight gained into this problem by the present renewed discussion, this mistake seems likely to have been a useful rather than a harmful one.

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Comment on "A Finite-Element Approach for Nonlinear Panel Flutter"

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A USEFUL attempt to employ a finite-element method to study the nonlinear effects due to midplane stretching on panel flutter has been made by Mei.[1] However, the following reservations have to be made about the analysis in Ref. 1.

Following Mei,[1] for a two-dimensional flat plate of length a, thickness h, and mass per unit area m, the differential equation of motion is written as

$$D\frac{\partial^4 w}{\partial x^4} - (N_x + N_{x0})\frac{\partial^2 w}{\partial x^2} + m\frac{\partial^2 w}{\partial t^2} = p \qquad (1)$$

where

$$N_x = \frac{Eh}{2a}\int_0^a \left(\frac{\partial w}{\partial x}\right)^2 \mathrm{d}x \qquad (2)$$

is the membrane force induced by large deflection, N_{x0} is the initial in-plane loading (tension positive), and $D = Eh^3/12(1-\nu^2)$ is the bending rigidity. The first observation regards the manner in which the axial loading N_{x0} is introduced. If it is obtained by an initial relative displacement between the two edges so that these edges remain immovable subsequently, the membrane force N_x is as given in Eq. (2) above. However, if N_{x0} is obtained by the action of an applied load at the edge, so that relative movement between the edges is possible (i.e., movable) during vibration, then $N_x = 0$, i.e., no membrane action due to stretching will occur within the first order nonlinear theory considered here.

The stiffness equation of motion for Eq. (1) in Ref. 1 is based on Refs. 2 and 3 and it contains an error in the evaluation of the nonlinear stiffness due to the membrane force. Note that Eq. (2) requires a constant membrane force N_x over the length of the plate $x=0$ to $x=a$, whereas Eq. (7) of Ref. 1 requires that N_x be a constant within an element of length l, depend only on the element modal displacements $\{u_e\}$, and therefore that it vary from element to element. This is obviously based on an erroneous assumption that for an element lying between $x=x'$ and $x=x'+l$, the stretching force is given by

$$N_x' = \frac{Eh}{2l}\int_{x'}^{x'+l} \left(\frac{\partial w}{\partial x}\right)^2 \mathrm{d}x \qquad (3)$$

This describes an entirely different model from that described by Eq. (2). In other words, Eq. (2) describes a realistic model in which an in-plane longitudinal deformation is allowed, whereas Eq. (3) requires that no in-plane deformation be permitted and that each point on the panel have only a vertical deflection. The analysis in Ref. 1 can be improved by the author by redefining N_x so that it now takes into account the nature of Eq. (2) and therefore depends on the system nodal displacements $\{u\}$ and does not vary from element to element according to the element nodal displacements $\{u_e\}$ as in Eq. (7).

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Reply by Author to G. Prathap

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THE author would like to thank Prathap for his interest in the work and for his relevant remarks. He is correct in the assumption of immovable edge conditions, which are employed widely in nonlineear fluttering panels. However, the author would like to add that the axial loading N_{xo} could also be due to change in temperature. The author is also thankful to Prathap for bringing out a few clarifications in the assumptions on membrane force N_x. The author is now modifying the computer program, and new results will be presented later.

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Errata

Modeling of Convective Mode Combustion through Granulated Propellant to Predict Detonation Transition

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THE total energy per unit mass e for both the gas phase and solid phase should include the kinetic energy as well. Thus, in Eq. (10),

$$e_g = c_{v_g} T_g + u_g^2/2$$

and in Eq. (11),

$$e_p = c_{v_p} T_p + u_p^2/2$$

This also means that the last term in each of the field balance energy equations (10) and (11), describing the gas-particle heat transfer, should read, respectively

$$\mp \bar{h}[3\alpha_2 (T_g - T_p)]/r_p$$

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